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A comparison of analytical and optimization inverse techniques for characterizing intrinsic parameters of porous materials

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In recent years, sound absorbing materials are finding many applications in transport Industry. In automobiles they are used as noise control treatments in engine compartment as well in passenger cabin. To understand and simulate acoustic behavior of these noise control treatments, intrinsic physical parameters are required. The acoustical behavior of poroelastic material is governed by five macroscopic intrinsic parameters e.g. porosity, flow resistivity, tortuosity and characteristics lengths as well as three mechanical parameters. Out of these five physical parameters, porosity and flow resistivity can be measured directly by available standardized methods. But measurement of physical parameters like tortuosity, viscous and thermal characteristic lengths is very difficult and no accepted procedure is available for their measurement. As an alternative, analytical inverse approach (mid-frequency) and optimization technique Genetic algorithm are well known in the literature. This paper compares the results from both techniques for intrinsic parameters. It also presents the effect of inverted parameters on sound absorption of the porous materials using Johnson-Champoux-Allard model.

1 INTRODUCTION

Automotive sound packages consist of open cell acoustic poroelastic materials for noise control purposes. The optimization of such systems depends on the knowledge of intrinsic physical parameters of the porous materials, but only few parameters viz. porosity and flow resistivity can be measured directly using available test methods while measurement of physical parameters like tortuosity, viscous and thermal characteristic lengths is very difficult. So as an alternative, many authors have proposed different inversion strategies for getting these properties from directly measured both characteristic and surface properties of the material using standing wave tube. These inverse characterization schemes are based on the equivalent fluid model (e.g. Johnson-Champoux-Allard) in which the solid frame is assumed to be rigid, i.e. motionless. The inverse characterization of the parameters is performed over a wide frequency range [50-4200 Hz] in which all these macroscopic physical parameters are dominant. At low frequencies, flow resistivity is very dominant while at mid frequencies, porosity, tortuosity and characteristic

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lengths are dominating. In the following sections, a description of the equivalent fluid model is presented followed by discussion on the inverse problem strategies and inverse characterization results for four porous samples. The paper also presents the effect of physical parameters estimated by inversion technique on acoustical properties using Johnson-Champoux-Allard (JCA) model.

2 THE EQUIVALENT FLUID: JOHNSON-CHAMPOUX-ALLARD MODEL

Open cell Poroelastic materials are very well described by Biot theory[1]. At the same time, in many situations when a material sample is excited by acoustical waves, the frame of this material behaves approximately as acoustically rigid (motionless) over a wide range of frequencies. In this case, the porous material can be replaced on a macroscopic scale by an equivalent fluid of effective dynamic density and effective bulk modulus. The motionless frame condition can occur either because of high density or elasticity modulus, or because of particular boundary conditions imposed during the test. The dynamic density $\rho(\omega)$ and complex bulk modulus $K(\omega)$ for Johnson model are given by following equations[2,12].

$$\rho\left(\omega\right)=\rho_0\alpha_\infty\left[1+\frac{\sigma\varphi}{j\omega\rho_0\alpha_\infty}\sqrt{\frac{4j\alpha_\infty{}^2\eta\omega}{\sigma^2\Lambda^2\varphi^2}}\right] \quad (1)$$

$$K\left(\omega\right)=\gamma P_0\left[\gamma-\left(\gamma-1\right)\Big/1+\frac{8\eta}{j\Lambda' N_{pr}\omega\rho_0}\sqrt{1+j\rho_0\frac{\omega N_{pr}\Lambda'}{16\eta}}\right]^{-1} \quad (2)$$

For a porous sample of thickness d, backed by a rigid wall, specific acoustic surface impedance is given as

$$Z_S=-j\frac{Z_C}{\phi}\cot\left(k_C\, d\right) \quad (3)$$

The characteristic impedance and the complex wave number of the porous specimen are related to the effective properties of the porous medium by Eqns. (4) and (5)

$$Z_C=\left(\rho\left(\omega\right)K\left(\omega\right)\right)^{1/2} \quad (4)$$

$$k_C=j\omega.\left(\rho(\omega)/K(\omega)\right)^{1/2} \quad (5)$$

Using Eqn. (3), sound absorption coefficient of a porous material is given by

$$\alpha=1-\left|\left(Z_S-\rho_0c_0\right)/\left(Z_S+\rho_0c_0\right)\right|^2 \quad (6)$$

3 ANALYTICAL METHOD

Analytical equations formulated by inverting JCA model can be used to derive macroscopic physical parameters from the dynamic density $\rho(\omega)$ and complex bulk modulus $K(\omega)$ in sound absorbing media. The methods using this approach are also known as indirect methods. These methods use low frequency approach for deriving flow resistivity from imaginary part of dynamic density. With prior knowledge of flow resistivity and porosity; tortuosity and characteristics lengths can be estimated from mid or high frequency approach using dynamic density and dynamic bulk modulus of the material.

3.1 Determination of Flow Resistivity

In Johnson-Champoux-Allard model, the dynamic density is given by Eqn. (1) which is a complex valued term. After applying some mathematics and decomposing Eqn. (1) in real and imaginary parts, one can obtain[3]

$$\rho(\omega) = \left(\frac{\rho_0 \alpha_\infty}{\varphi} + \frac{\sigma}{\omega} G_I \right) - \frac{\sigma}{\omega} G_R \qquad (7)$$

Where $G_R = \frac{1}{\sqrt{2}} \sqrt{1 + \sqrt{1 + \left(\frac{M \varpi}{2} \right)^2}}$ and $G_I = \frac{M\varpi}{4G_R}$ with $M = \frac{8\alpha_\infty \eta}{\phi \sigma \Lambda^2}$ and ϖ is the given by ω / ω_v where ω_v is the viscous characteristic frequency defined by Biot and estimated by $\omega_v = \sigma\phi / \rho_0 \alpha_\infty$. From Eqn. (7), the low frequency limit of the dynamic density is given as $\left(-\sigma / \omega \right)$.

$$\sigma = - \lim_{\omega \to 0} \left[\operatorname{Im}\left(\rho(\omega) \right) . \omega \right] \qquad (8)$$

The imaginary part of the low frequency limit of the dynamic density $\rho(\omega)$ is the flow resistivity[4]. It is important to mention that JCA model fails to predict reality at low frequency, especially regarding the real part of dynamic density which is underestimated when $\omega << \omega_v$. So a mid frequency approach is proposed here for the determination of tortuosity and viscous characteristic length. These parameters including thermal characteristic length are estimated in the frequency range [1100 ; 1800] Hz.

3.2 Determination of Geometrical Tortuosity

With prior knowledge of flow resistivity and porosity and using simplified equation for dynamic density, tortuosity can be calculated as

$$\alpha_\infty = \frac{\varphi}{\rho_0} \left(\operatorname{Re}\left(\rho(\omega) \right) - \sqrt{ \operatorname{Im}\left(\rho(\omega) \right)^2 - \frac{\sigma^2}{\omega^2} } \right)$$

(9)

Tortuosity is frequency independent[5].

3.3 Determination of Viscous Characteristic Length (VCL)

With prior knowledge of flow resistivity, porosity and tortuosity, and using simplified equation for dynamic density, viscous characteristic length can be calculated as

$$\Lambda = \frac{\alpha_\infty}{\varphi} \left(\frac{2\eta\rho_0}{\omega \operatorname{Im}\left(\rho(\omega) \right) \left(\alpha_\infty \rho_0 / \varphi - \operatorname{Re}\left(\rho(\omega) \right) \right)} \right)$$

(10)

Viscous characteristic length is frequency independent [5].

3.4 Determination of Thermal Characteristic Length (TCL)

A similar approach as discussed in earlier sections can be used for the determination of thermal characteristic length. For the determination of thermal length, imaginary part of bulk modulus from the Lafarge's Model[6] can be used. In Lafarge's Model, one more additional parameter, thermal permeability is used. But the thermal length is only governing parameter for thermal dissipation in JCA model.

$$\Lambda' = 2\sqrt{\frac{\kappa}{\rho_0 \gamma \omega}} \left(-\operatorname{Im}\left(\left(\frac{\gamma P_0 - \varphi K(\omega)}{\gamma P_0 - \gamma \varphi K(\omega)} \right)^2 \right) \right)^{-1} \tag{11}$$

Thermal characteristic length is also frequency independent.

4 OPTIMIZATION BASED METHOD: GENETIC ALGORITHM

Genetic algorithm is based on the Darwin's theory of Evolution. It is used to solve the optimization problem with constraints and bounds on the solution. It repeatedly modifies a population of individual points using rules modeled on gene combinations in biological reproduction. At each step, the genetic algorithm selects individuals at random from the current population to be "parents" and uses them produce the "children" for the next generation. Over successive generations, the genetic algorithm improves the chances of finding a global solution. In the final analysis, normalized surface impedance is used as cost function. The cost function minimized is defined as Eqn. (7)

$$Z_S = \sum \left| Z_{s,Mea} - Z_{s,Model} \right| \tag{12}$$

The bounds implemented on the physical parameters are given in the Table 1

Also non-linear bound was implemented on characteristics lengths such that $\Lambda \leq \Lambda'$. This condition is true for almost all porous materials. The optimization problem with constraints was implemented in Matlab®.

5 EXPERIMENTAL METHODS

For Experimental measurements, four different types of porous as well fibrous materials like Melamine foam, Polyurethane foam, Soft Felt and Kenaf were selected with density in between 20 and 40 kg/m^3 and thickness in between 20 and 30 mm. The diameter of all samples was 45 mm. Figure 1 shows samples used for inverse characterization. The open porosity was directly measured by a method based on Boyle's law[7] which uses isothermal compression of air volume within and external to the tested material. The static flow resistivity was measured by flow resistivity test rig based on standard ISO-9053[8]. Finally, the tortuosity was determined by a method based on determination of the high frequency limit for the complex phase velocities within the air and the material[9]. While the characteristic lengths were inverted using Genetic algorithm with directly measured porosity, flow resistivity, tortuosity as additional input to Genetic Algorithm. The directly measured physical material parameters are tabulated in the Table 2. Afterwards the surface acoustic properties (i.e. surface impedance and the normal incidence sound absorption coefficient) were measured according to the ISO 10534-2[10]. Finally, the normal incidence sound transmission loss of similar 100 mm samples was measured in four microphone tube by means of a transfer matrix approach[11].

6 RESULTS AND DISCUSSIONS

The macroscopic physical parameters estimated from analytical and Genetic inverse techniques are compared in Table 2. In Genetic inverse characterization all five physical parameters are inverted from measured surface impedance from impedance tube, while in analytical inverse only four parameters are estimated. Flow resistivity is estimated from low frequency limit (below 800 Hz) of dynamic density which can be measured using three or four microphone tube methods given in the literature and it compares well with flow resistivity calculated from Genetic optimization as well as with experimental values. In analytical inverse, tortuosity and characteristic lengths are estimated at mid frequency range of complex dynamic density using Eqns. (9) and (10) with estimated value of flow resistivity and measured porosity. The tortuosity calculated for materials like glass wool and felt is near to 1, which is classical result for fibrous materials. There is good agreement between tortuosity measured directly, estimated by analytical inverse technique and values calculated by Genetic optimization as seen from Table 2. Figures 2(a) and 2(b) show that tortuosity and viscous length are relatively constant at mid frequency range. The dynamic bulk modulus is used for calculation of thermal length using Lafarge's model. It is also found to be constant for materials at mid frequency range. The frequency dependency of tortuosity and viscous length calculated for polyurethane foam is shown below in Figs. 2(a) and 2(b) respectively.

The estimated thermal length is shown in Fig. 3(a) and it seems to be relatively constant over mid frequency range. To check the effect of estimated parameters on sound absorption and surface properties, predictions given by Johnson-Champoux-Allard model using measured parameters are compared with the initial measurements. The comparison of the predicted surface impedance using five parameters inverted from Genetic optimization with that of experimental surface impedance is shown in Fig. 3(b), which shows good agreement in measurement and prediction. The effect of estimated parameters from both the inverse techniques on normal sound absorption is studied and results are shown in Figs. 4(a) and 4(b) for melamine foam and polyurethane foam respectively. There is good agreement between the measurement and prediction of sound absorption coefficient except in Fig. 4(a) in which there is slight deviation in measurement from prediction at 2000 Hz, it is due to structural resonance of the frame as melamine foam is poro-elastic and the frame gets excited by sound waves during the measurement of sound absorption coefficient. Johnson-Champoux-Allard model can not predict elastic behavior of poroelastic materials as seen from Fig. 4(a). The comparison of predicted dynamic density and bulk modulus with estimated parameters using analytical inverse characterization with experimental measured values of dynamic density and bulk modulus is shown in Figs. 5(a) and 5(b). Similarly in Fig. 5(b), the frame resonance is dominant around 3000 Hz. As seen from these figures, it is clear that, the macroscopic physical parameters give very good correlation with measured surface properties and sound absorption coefficient so these physical parameters can be used for prediction of sound absorption coefficient and sound transmission loss of the materials.

7 CONCLUSIONS

The paper presents a comparison of analytical inverse method and Genetic optimization based inverse technique using complex acoustical properties. From this comparison it is clear that, it is possible to characterize at least four physical intrinsic parameters viz. flow resistivity, tortuosity, viscous and thermal characteristics lengths with adequate accuracy using analytical inverse while all five physical parameters using optimization based inverse technique. It is also possible to use analytical method to cross check the physical intrinsic parameters inverted from Genetic optimization based inverse technique. It is also found that for homogenous materials tortuosity and characteristic lengths are almost stable over the mid frequency range.

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Table 1 - Bounds on Physical Parameters.

Bounds	σ	ϕ	α_∞	Λ	Λ'
Lower Bounds	1000	0.1	1	10	10
Upper Bounds	200000	1	10	2000	2000

Table 2 - Comparison of Intrinsic Physical Parameters.

Melamine Foam 8.8 Kg/m^3- 29.4 mm					
Parameters	σ	ϕ	α_∞	Λ	Λ'
Exp	10518	0.99	1.01	107	137
Analytical	10634	0.98	1.00	99	180
Optimization	10872	0.99	1.00	99	142

Polyurethane Foam 40 Kg/m^3- 40 mm					
Parameters	σ	ϕ	α_∞	Λ	Λ'
Exp.	5359	0.98	1.10	48	240
Analytical	6036	0.98	1.11	125	295
Optimization	6298	0.99	1.14	135	244

Soft Felt 24 Kg/m^3- 25 mm					
Parameters	σ	ϕ	α_∞	Λ	Λ'
Exp.	6114	0.99	1.03	140	230
Analytical	6135	0.99	1.00	164	230
Optimization	5931	0.99	1.02	165	294

Kenaf 40 Kg/m^3- 20 mm					
Parameters	σ	ϕ	α_∞	Λ	Λ'
Exp.	6215	0.99	1.05	68	177
Analytical	6918	0.98	1	62	185
Optimization	6731	0.99	1.01	67	180

			
Melamine - 29 mm	PU-Foam 25mm	Soft Felt- 25 mm	Kenaf -20 mm

Fig. 1 - Samples of Materials used for Measurement of physical parameters.

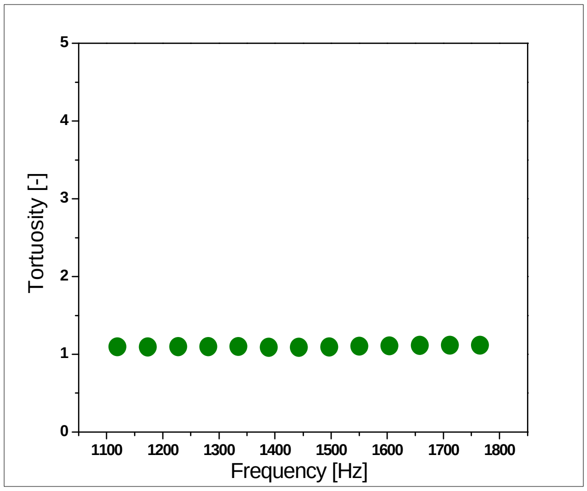


Fig. 2(a) - Estimated tortuosity (α_∞) *for PU-foam.*

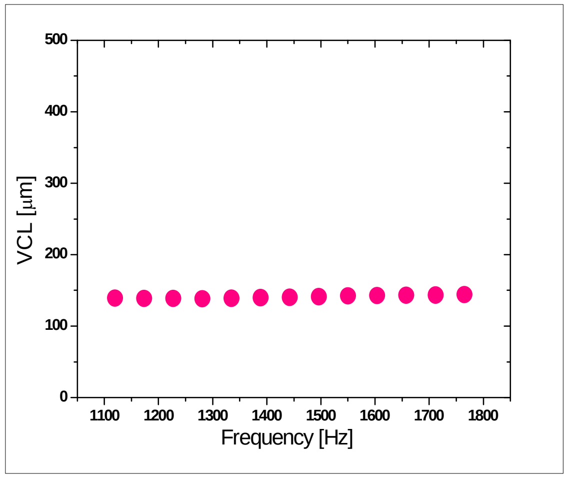
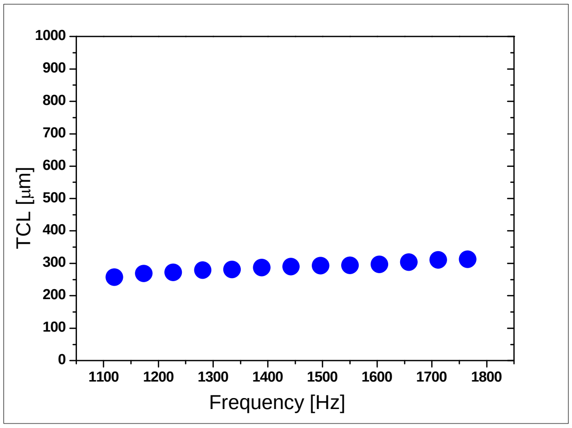


Fig. 2(b) - Estimated Viscous characteristic length (Λ) *for PU-foam.*



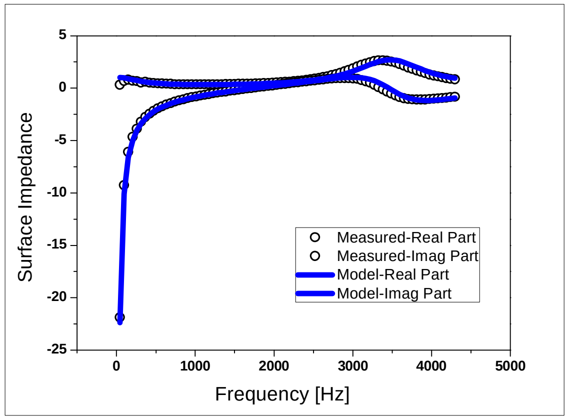
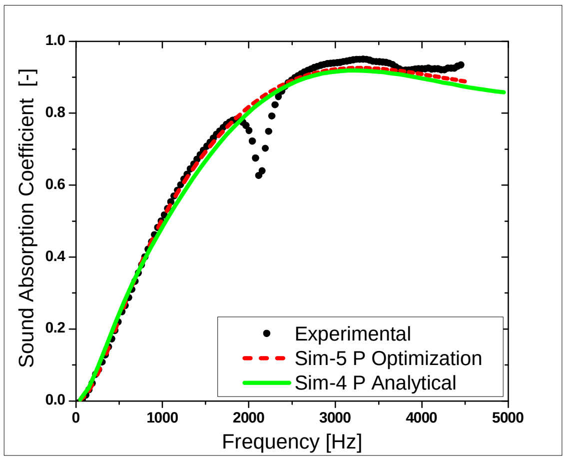


Fig. 3(a) - Computed Thermal characteristic length $\left(\Lambda'\right)$ for PU-foam.

Fig. 3(b) - Comparison of measured and predicted Surface impedance for PU-Foam.



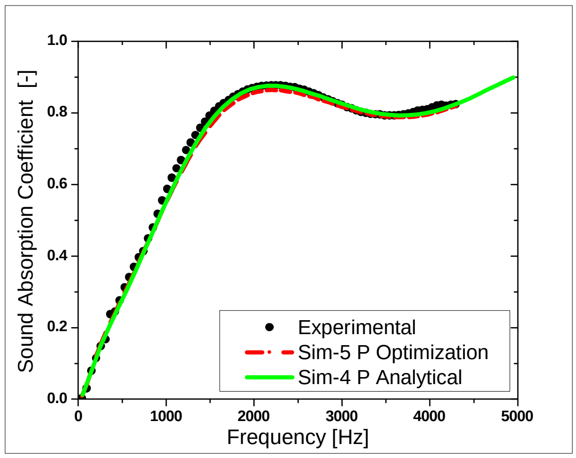


Fig. 4(a) - Comparison of measured and predicted Sound absorption coefficient for Melamine Foam.

Fig. 4(b) - Comparison of measured and predicted Sound absorption coefficient for Polyurethane Foam.

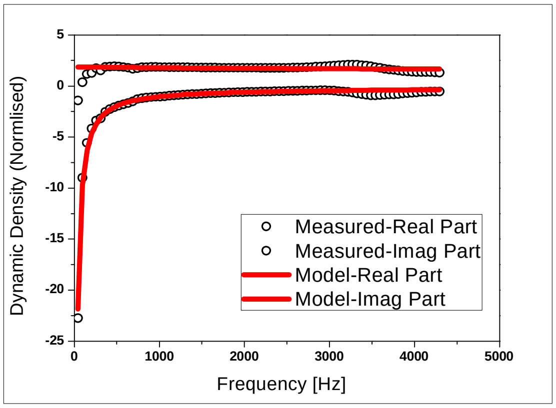


Fig. 5(a) - Comparison of measured and predicted dynamic density of for PU-foam

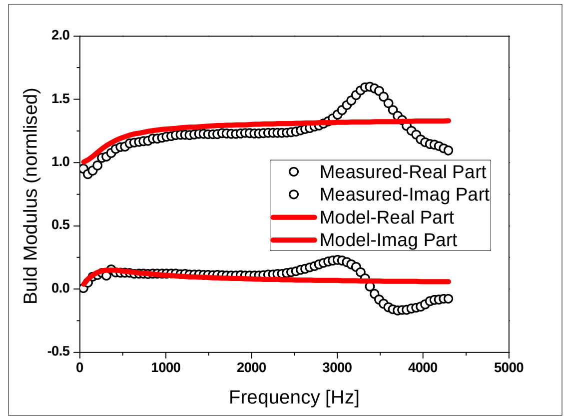


Fig. 5(b) - Comparison of measured and predicted bulk modulus of PU-foam